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Phase Shift Determination of Imperfect Open Calibration Standards

Gary Biddle

Abstract —**A new measurement technique for determining the inherent phase shift of open calibration standards for network analyzers due to fringing capacitance is presented. The resultant phase shift is directly measured using an uncalibrated network analyzer and requires no modeling of coefficients of capacitance as conventional methods do. An exact expression for the phase shift of an imperfect open is derived for each frequency point. Two sets of standard one-port error equations are developed for the application. The traditional set of calibration standards, the match, short, and imperfect open, are used. The standards are measured twice: once at the reference plane and then offset by a precision piece of air line. Results are presented for the phase shifts of a few open calibration standards at discrete frequencies.**

I. Introduction

Network analyzers have been used extensively to characterize microwave components and devices for several decades. Improvements in instrumentation hardware, computer availability, and new calibration standards and procedures have enhanced measurement capabilities.

Initially, with the formulation of signal flow graphs well documented, early works by Hackborn [1] and Hand [2] introduced the automatic network analyzer system. Attention focused on hardware description, calibration procedures, and measurement accuracy. The error models appearing in these works were eight-term with the following calibration standards: the match, the short, and the offset short.

A few years later, an open calibration standard was introduced as an option to the offset short by Kruppa [3]. By 1978 it was known and pointed out in works by Rehnmark [4], daSilva and McPhun [5], and Fitzpatrick [6] that opens were imperfect because of radiation and stray capacitance.

The phase shift of an imperfect open was addressed by daSilva and McPhun. The measurement procedure required four test pieces with identical terminations, identical propagation factors for offsets of prescribed lengths, and a short circuit test piece. A total of five measurements were required.

Hewlett Packard approached the phase shift problem of an imperfect open in a different manner. In Application Note 221A, an accuracy enhancement program using coefficients of capacitance to correct for the residual fringing effects of a shielded open was presented. The resultant phase shift was modeled as a function of frequency. The coefficients of capacitance were then chosen to best fit the selected measurement responses.

In this paper, a new measurement technique that obtains the phase shift directly is presented. The conventional calibration standards are used: the match (fixed and/or sliding load), the short, and the imperfect open. One piece of precision air line is also required.

In contrast to prior measurement procedures, no special test pieces are needed, no identical terminations or propagation factors for prescribed lengths are required, and no coefficients of capacitance are required.

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II. Error Equations

This section shows the two sets of error equations needed to determine the phase shift of the imperfect open. The conventional methods of signal flow analysis, using Mason's rule, are employed. The well-known one-port error network is obtained.

In order to determine the open's phase shift, a total of six measurements must be made. The first set of measurements require the three standards to be measured at a reference plane. The error terms of the reference plane are unknown. Thus it is an uncalibrated measurement.

The second set of measurements require the three standards to be measured again at the same reference plane but offset with a precision piece of air line. Again the error terms are unknown and the measurement is uncalibrated. The introduction of the air line into the second set of measurements has added an additional propagation factor which is unknown.

The reflection coefficients of the calibration standards and the propagation factor of the air line appear in the error network diagrams. The three reflection coefficients are treated as follows.

The match in the ideal case is reflectionless, thus having a reflection coefficient of zero. The reflection coefficient of the match is represented as zero in the error equations. The standard practice of utilizing the sliding load at higher frequencies to enhance the measurement of the true system directivity is used.

The short in the ideal case reflects all the incident energy with a phase inversion at all frequencies. The reflection coefficient of the short is represented as $|1|$ with an argument of π in the error equations. The precision shorts found in 3.5 mm, 7 mm, and 14 mm calibration kits have very low residual inductance; thus they may be considered ideal for this measurement technique.

The open in the ideal case reflects all the incident energy with no phase shift. In practice, there is an appreciable phase shift associated with an open. The reflection coefficient of the imperfect open is represented as $|1|$ with an unknown argument in the error equations. Thus only four measurements can be made of known calibration standards.

The precision piece of air line is required to offset the calibration standards. The air line is considered to be reflectionless with unknown propagation factors and length. It is depicted as such.

The flow graphs for the two sets of error equations are shown in Fig. 1 and Fig. 2. The upper flow graph depicts the standard one-port error network. The lower flow graph depicts a one-port error network which includes an additional offset.

The variable Γ_m is the reflection coefficient measured at the analyzer's measurement plane, while Γ_{cs} represents the reflection coefficient of the calibration standard applied at the reference plane. In the conventional way, the three error terms $\exp(-kl)$ represents the offset introduced by the reflectionless air line.

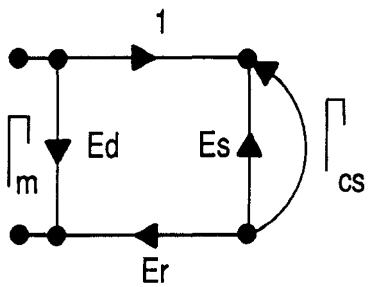

Fig. 1. Standard one-port model. Flow diagram for the first set of measurements.

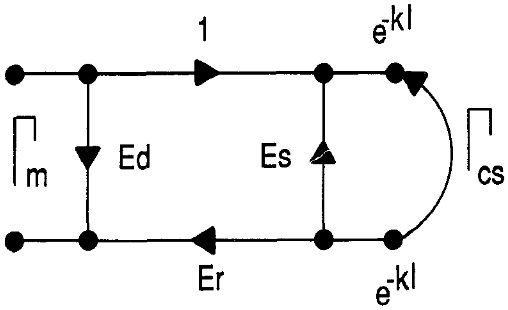

Fig. 2. One-port model with offset. Flow diagram for the second set of measurements.

Using Mason's rule, the reflection coefficient Γ_m can be expressed as a function of the three error terms, the reflection coefficient of the calibration standard, and $\exp(-kl)$ for the offset case.

For the standard one-port error model the error equation is

$$\Gamma_m = Ed + \frac{Er\,\Gamma_{cs}}{1 - Es\,\Gamma_{cs}} \tag{1}$$

For the one-port model with offset the error equations is

$$\Gamma_m = Ed + \frac{\exp(-kl)\,Er\,\Gamma_{cs}}{1 - \exp(-kl)\,Es\,\Gamma_{cs}} \tag{2}$$

The conventional way is to consider the three error terms in the above equations as unknowns. However, since Ed can be measured directly using the matched standard, the substitution $\Gamma'_m = \Gamma_m - Ed$ will be made. This will allow the error equations to be expressed in matrix form. Applying the known reflection coefficient for a short and the unknown reflection coefficient for a shielded open to both sets of error network equations, the matrix representation is as follows.

Matrix equation for the standard one-port error model is

$$\begin{bmatrix} e^{i\pi} & e^{i\pi}\Gamma'_{ms} \\ e^{i\theta} & e^{i\theta}\Gamma'_{mo} \end{bmatrix} \begin{bmatrix} Er \\ Es \end{bmatrix} = \begin{bmatrix} \Gamma'_{ms} \\ \Gamma'_{mo} \end{bmatrix} \tag{3}$$

where

$e^{i\theta}$ = the unknown phase shift for the open standard,
Γ'_{ms} = the measured short reflection coefficient,
Γ'_{mo} = the measured open reflection coefficient.

The matrix equation for the one-port model with offset is

$$\begin{bmatrix} e^{i\pi} & e^{i\pi}\Gamma'_{mso} \\ e^{i\theta} & e^{i\theta}\Gamma'_{moo} \end{bmatrix} \begin{bmatrix} Er \\ Es \end{bmatrix} = \begin{bmatrix} \Gamma'_{mso} & e^{2kl} \\ \Gamma'_{moo} & e^{2kl} \end{bmatrix} \tag{4}$$

where

$2kl$ = the propagation term for the air line,
Γ'_{mso} = the measured short offset reflection coefficient,
Γ'_{moo} = the measured open offset reflection coefficient.

The inverse matrices of the error term coefficients matrices do exist. Multiplying through by the corresponding inverse coefficient matrix, the unknown error terms, Er and Es, may be equated from the two sets of matrix equations. Thus the unknown error terms are removed and the network analyzer is consider to be uncalibrated for these measurements.

Solving for the inverse matrices and equating terms, the following equation is obtained:

$$\begin{bmatrix} -\Gamma'_{mo}/(\Gamma'_{ms}-\Gamma'_{mo}) & \Gamma'_{ms}/(\Gamma'_{ms}-\Gamma'_{mo}) \\ 1/(\Gamma'_{ms}-\Gamma'_{mo}) & -1/(\Gamma'_{ms}-\Gamma'_{mo}) \end{bmatrix} \begin{bmatrix} \Gamma'_{ms}e^{-i\pi} \\ \Gamma'_{mo}e^{-i\theta} \end{bmatrix}$$
$$= \begin{bmatrix} -\Gamma'_{moo}/(\Gamma'_{mso}-\Gamma'_{moo}) & \Gamma'_{mso}/(\Gamma'_{mso}-\Gamma'_{moo}) \\ 1/(\Gamma'_{mso}-\Gamma'_{moo}) & -1/(\Gamma'_{mso}-\Gamma'_{moo}) \end{bmatrix}$$
$$\cdot \begin{bmatrix} \Gamma'_{mso}e^{2kl}e^{-i\pi} \\ \Gamma'_{moo}e^{2kl}e^{-i\theta} \end{bmatrix}. \tag{5}$$

Performing the matrix multiplication, substituting for $e^{-i\pi}$, and collecting terms, the following two equations with two unknowns are left. The two unknowns are the phase shift of the imperfect open and the propagation term for the air line:

$$(\Gamma'_{mo}\Gamma'_{ms}/(\Gamma'_{ms}-\Gamma'_{mo})) = (\Gamma'_{moo}\Gamma'_{mso}/(\Gamma'_{mso}-\Gamma_{moo}))e^{2kl} \tag{6}$$

$$(\Gamma'_{ms}/(\Gamma'_{ms}-\Gamma'_{mo})) + (\Gamma'_{mo}/(\Gamma'_{ms}-\Gamma'_{mo}))e^{-i\theta})$$
$$= [(\Gamma'_{mso}/(\Gamma'_{mso}-\Gamma'_{moo}))$$
$$+ (\Gamma'_{moo}/(\Gamma'_{mso}-\Gamma'_{moo}))e^{-i\theta}]e^{2kl}. \tag{7}$$

By equating e^{2kl} in (6) and (7), the unknown properties of the air line are removed. After collecting terms, the following expression involving the unknown phase shift and measured quantities is obtained:

$$A + Be^{-i\theta} = 0 \tag{8}$$

where

$$A = (\Gamma'_{mo}\Gamma'_{ms}\Gamma'_{mso} - \Gamma'_{ms}\Gamma'_{moo}\Gamma'_{mso}) \tag{9}$$

$$B = (\Gamma'_{mo}\Gamma'_{ms}\Gamma'_{moo} - \Gamma'_{m0}\Gamma'_{moo}\Gamma'_{mso}). \tag{10}$$

By equating the real and imaginary parts of (8), two expressions for the phase shift of an imperfect open are obtained:

$$\theta = \cos^{-1}\left[\mathrm{Re}\,\frac{-A}{B}\right] \tag{11}$$

and

$$\theta = -\sin^{-1}\left[\mathrm{Im}\,\frac{-A}{B}\right]. \tag{12}$$

If the real and imaginary parts of A and B are expressed in terms of the measured Γ, it can be shown that the numerators in (11) and (12) are not equal and likewise for the denominators. Numerically, both expressions are used and the average is then taken for the phase shift.

Once the phase shift is determined, the true reflection coefficient of the open is precisely known. The phase shift can then be used in any calibration procedure requiring this open. Instead of modeling an error network including an ideal open, one models an error network including an imperfect open with its respective phase shift.

For the case of the match, short, and open calibration procedure for network analyzers, the measured phase shift may be directly substituted into the equations determining the error

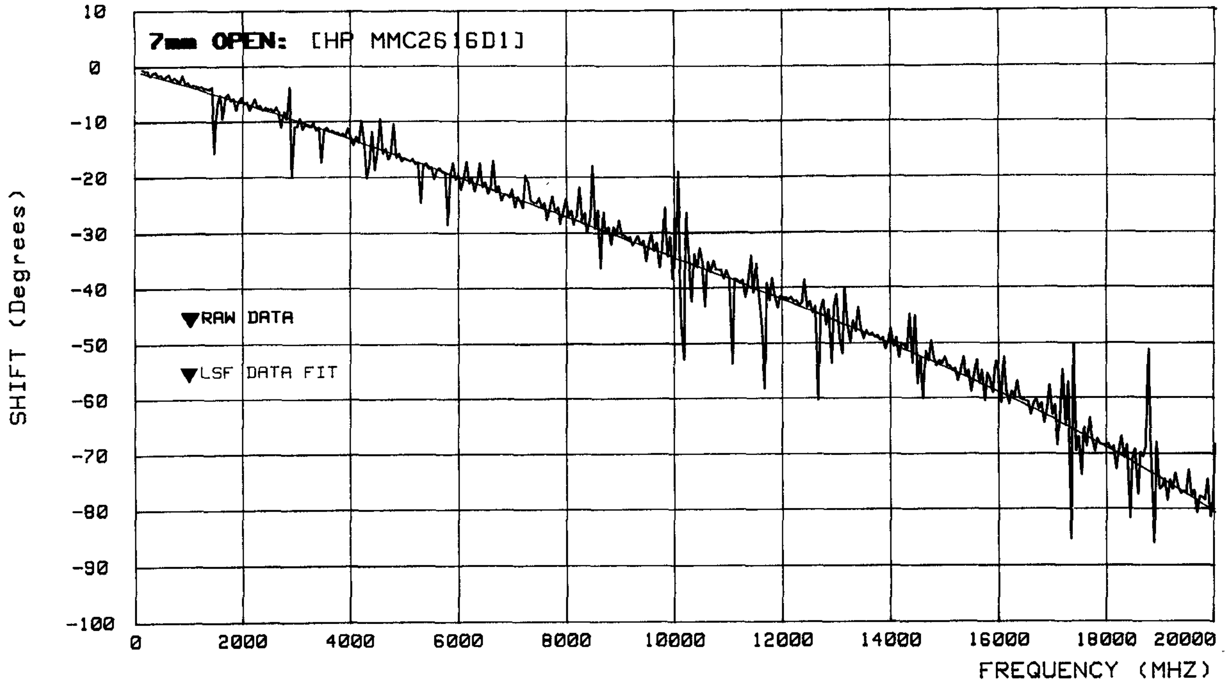

Fig. 3. Correction factor for 7 mm open.

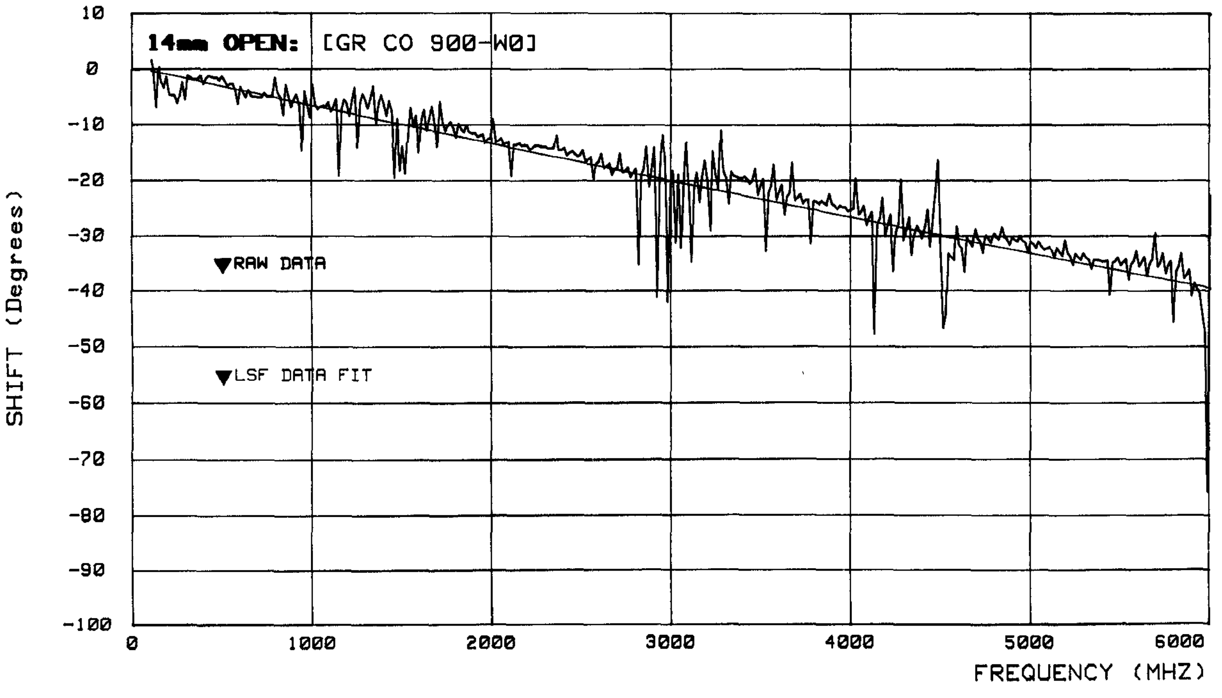

Fig. 4. Correction factor for 14 mm open.

terms. In particular, it would be most useful in cases where the coefficients of capacitance were unknown or prototype opens for special measurement conditions were used. Assuming the directivity error term, Ed, is not an unknown because it can be directly determined by measurement, the error terms Er and Es can be expressed as follows:

$$\begin{bmatrix} Er \\ Es \end{bmatrix} = \begin{bmatrix} -1 & -\Gamma'_{ms} \\ e^{i\theta} & e^{i\theta}\Gamma'_{mo} \end{bmatrix}^{-1} \begin{bmatrix} \Gamma'_{ms} \\ \Gamma'_{mo} \end{bmatrix}$$

where

$e^{i\theta}$ = the phase shift of an imperfect open,
Γ'_{ms} = the measured Γ for a short at the reference plane,
Γ'_{mo} = the measured Γ for an open at the reference plane.

This representation of these error terms applies to all standard error equations using the match, short, short, and open calibration procedure. The above is also valid for the full two-port [12 error terms] error correction procedure.

III. Results for 7 mm Open Standard

The above method was used on an open calibration standard that accompanies HP8510 network analyzers. The 7 mm open, part number MMC2616D1, was evaluated from 0.1 to 20 GHz. A piece of 10 cm air line, HP11566A with support beads, was used to offset the calibration standards.

Measurements were first made on the three standards at a reference plane. A fixed load was used between 0.1 and 2 GHz, with a sliding load used at the higher frequencies. A machine averaging factor of 1024 was used to improve measurement accuracy. These measurements were repeated a second time, in which the three standards were offset by the piece of air line.

The measured phase shift of the imperfect open is shown in Fig. 3. The raw data are presented along with a least square fit of the data. Increased averages and repeated acquisitions can improved the raw data. The true phase shift is a smooth function. The least square fit of the data yields the exact correction factor needed for the open. A detailed printout of the phase shift of the 7 mm open is available on request.

IV. Application of Method

This approach was found most useful in a measurement study of coaxial discontinuities. An investigation was done to determine the axial separation distance of the inner and outer step discontinuities which produced minimum reflections. To facilitate fabrication of test pieces, a 7 mm to 14 mm type transition was studied. This required a calibration procedure using a 14 mm prototype open.

Measurements of the open's phase shift for the 14 mm open are presented in Fig. 4. Again the least square fit of the raw data was used to determine the phase shift factor. The phase shift factor was used to determine the error terms, *Er* and *Es*, needed in the measurement. A detailed printout of the phase shift for the 14 mm open is available on request.

V. Conclusions

This empirical method can be used to accurately determine the correction factor needed for imperfect opens due to fringing capacitance. It offers the advantage of determining the unknown phase shift for an open by using the same unknown open, an air line with unknown properties, and a network analyzer which is not calibrated. No other calibration kits or modeling coefficients are necessary. This method can be used with opens of any type if an accompanying short, match, and air line exist.

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An Analytical Approach to the Analysis of Dispersion Characteristics of Microstrip Lines

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Abstract—**A new analytical method for determining the dispersion characteristics of microstrip lines is given. The method uses dual integral equations, and the dispersion relation is obtained in terms of a double infinite system of linear equations with good convergence properties.**

I. Introduction

Microstrip is one of the most important elements in microwave integrated circuits and microwave networks. In the early stage of microstrip-line analysis, much of the work was based on the quasi-TEM approximation [1]–[4]. This approximation is valid only for low frequences, and the resulting parameters, such as characteristic impedance and the propagation wavenumber, are independent of frequency. However, this approximate model is inadequate for estimating the dispersion properties of the microstrip line at higher frequences; consequently, a more rigorous full-wave analysis is required [5]. Various methods have been employed to calculate the dispersion characteristics of the stripline. Thus Hornsby and Gopinath [6] applied the finite difference method and a minimization technique. Dally [7] applied the finite element method; Zysman and Varon [8] formulated the integral equations of the problem; and Yamashita and Atuski [9] solved these integral equations numerically by nonuniform discretization of the integral domains. For shielded microstrip lines Mittra and Itoh [10] used the singular integral equation approach for deriving a new form of the dispersion equation with superior convergence properties. The spectral-domain approach has also often been applied to the full-wave analysis of the microstrip lines [11]–[14]. We also mention application to the microstrip problem of the variational conformal mapping technique [15].

Some of the developed methods are based on the assumption of certain "closed form" expressions for the longitudinal and transverse current distributions on the strip. As the proposed forms do not reveal the frequency and dielectric constant dependence of the current distributions with good accuracy, the results obtained with various methods have sometimes been quite different [16].

In this paper we developed a method to analyze the problem of microstrip shielded by two parallel planes similar to the method given by Mittra and Itoh [19] for the case of the completely shielded microstrip. Since in our case the dielectric domain is infinite, there follows a system of two integral equations instead of series equations corresponding to the bounded dielectric domain considered in [10]. We have succeeded in transforming the system of integral equations into an infinite system of linear equations. As a by-product, there follow two compatibility conditions which yield the dispersion equation of the problem.

II. Formulation of the Problem

In Fig. 1 the cross section of the microstrip line to be analyzed is shown. The geometry contains a conducting strip placed on a dielectric substrate and two perfectly conducting planes. The

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